IJIRR

International Journal of Information Research and Review
Vol. 06, Issue, 08, pp.6422-6428, August, 2019

REVIEW ARTICLE

MULTI-VAGUE SOFT SET AND ITS PROPERTIES

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ARTICLE INFO

Article History:

Received 14th May, 2019
Received in revised form
10th June, 2019
Accepted 25th July, 2019
Published online 30th August, 2019

Keywords:

Soft Set, Multi-fuzzy Set; Vague set; multi-Vague Soft Set.

ABSTRACT

Soft set and multi-fuzzy set have been successfully used as an effective mathematical tool for dealing with uncertainty. In this work, we first extend multi-fuzzy set to multi-vague set. By combining the multi-vague set and soft set models, we then introduce the concept of multi-vague soft sets, which can be regarded as a extension of some existing soft set models. We also define some operations on multi-fuzzy vague soft set and study some of their desirable properties. Furthermore, we discuss the lattice of multi-fuzzy vague soft set.

INTRODUCTION

There are a variety of uncertain problems in real world. Among of them can not be dealt effectively with using the existing tool, such as fuzzy set, interval mathematics, rough set and so on. For solving these problems, Molodsov (1) proposed the concept of soft set, which is a new mathematic tool for modeling uncertainties. Since its appearance, soft set theory has attracted more and more attention from many researchers and many results on soft set have been obtained in theory and application. For example, Maji and Biswas et al. (2) defined some basic operations of soft set. Ali et al. (3,4) further gave some new operations on soft sets and discussed some of their properties. Feng et al. (5,6) discussed the relations of fuzzy set and soft set, and presented some notions on how to combine themselves. Some researchers studied some algebraic structures based on soft set such as soft groups (7), soft rings (8,9), soft algebras (10) and soft BCK/BCI-algebras 11,12). In the research on soft set, the combination of soft set and other uncertain models (especially, fuzzy set and its extended models) is an important research direction. And lots of research results have been obtained. Maji et al. (13) first introduced the notion of fuzzy soft set by combining soft set and fuzzy set. Cagman and Karatag (14) further generalized fuzzy soft set to intuitionistic fuzzy soft set and applied it to decision making. Yang et al. (15) extended fuzzy soft set to interval-valued fuzzy soft set. By combining multi-fuzzy set and soft set, Yang et al. (16) proposed the concept of multi-fuzzy soft set. Some other combined models of soft set and their applications in decision making could be found in (17-23). Lately, Sebastian and Ramakrishnan (24) proposed the concept of the multi-fuzzy set, which is a new generalization of fuzzy set. Its membership function is an ordered sequence of traditional fuzzy membership functions. Multi-fuzzy set can deal with some problems which are hard to explain by other generalized fuzzy set models, such as color of pixels. In many real applications, however, due to the problems being complex and uncertain, it is difficult to represent the notions by exact numbers. In other words, it is more practical and reasonable to cope with these problems by intervals than certain single values. Therefore, the main goal of this paper is to combine vague set, multi-fuzzy set and soft set and obtain a new hybrid model called multi-vague soft set. It can be viewed as an interval-valued extension of the multi-fuzzy soft set or a generalization of the vague soft set. The rest of this paper is organized as follows. Section 2 briefly reviews some preliminaries. In Section 3, we extend multi-fuzzy set to multi-vague set, introduce some operations and discuss their properties. In Section 4, the concept of multi-vague soft set is first proposed by combining multi-vague set and soft set. Some operations on multi-vague soft set are defined and some of their properties are studied. Finally, Section 5 presents the conclusion.

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Preliminaries:

In this section, we shall briefly recall some basic notions being used in the study. Throughout this paper, let X be an initial universe of objects and Q the set of parameters in relation to objects in X. Let $P(X)$ denote the power set of X and $A \subseteq Q$. Molodtsov (1) first gave the definition of soft set as follows.

Definition 1. (1) A pair (F, A) is called a soft set over X, where $A \subseteq Q$ and F is a set valued mapping given by $F: A \to P(X)$. By combing fuzzy set and soft set, Maji et al. (13) introduced the notion of fuzzy soft set, which is a fuzzy extension of soft set.

Definition 2. (13) Let $\mathcal{P}(X)$ be the set of all fuzzy subsets of X. A pair (F, A) is called a fuzzy soft set over X, where F is a set valued mapping given by $F: A \to \mathcal{P}(X)$. From the above definition, we know that $F(e)$ is a fuzzy subset of X for each $q \in A$. As a new generalization form of fuzzy set, multi-fuzzy set was first introduced by Sebastian (24) as follows.

Definition 3. (24) Let n be a positive integer. A multi-fuzzy set $\widetilde{M}$ in X is a set of ordered sequences having the form
$\widetilde{M} = \{x/(\mu_1(x), \mu_2(x), \cdots, \mu_i(x), \cdots, \mu_n(x)): x \in X\}$,
where $\mu_i \in \mathcal{P}(X), i = 1,2, \cdots, n$.
The function $\mu_{\widetilde{M}} = (\mu_1, \mu_2, \cdots, \mu_n)$ is called the multi-membership function of multi-fuzzy set $\widetilde{M}$, n is called the dimension of $\widetilde{M}$. The set of all multi-fuzzy sets of dimension t n in X is denoted by $\mathcal{M}^n(X)$. Yang et al. (16) initiated the study on hybrid structures involving both multi-fuzzy sets and crisp soft sets. They proposed the following model called multi-fuzzy soft set, which is an extension of both crisp soft set and multi-fuzzy set.

Definition 4. (16) A pair (F, A) is called a multi-fuzzy soft set of dimension n over X, where F is a mapping given by $F: A \to \mathcal{M}^n(X)$.

Definition 5. (25) A vague set Z in the universe $X = \{x_1, x_2, \cdots, x_n\}$ can be expressed by $Z = \{x_i/\mu_Z(x_i) | x_i \in X\} = \{x_i/[t_Z(x_i), 1 - f_Z(x_i)] | x_i \in X\}$, and the condition $0 \leq t_Z(x_i) \leq 1 - f_Z(x_i)$ should hold for any $x_i \in X$, where $t_Z(x_i)$ and $f_Z(x_i)$ are called membership degree and nonmembership degree of element x_i to the vague set Z, respectively. The set of all vague sets on X is denoted by $\mathcal{V}(X)$.

Definition 6. (25) Let Y and Z be two vague sets on universe X. Then some operations of vague sets are given as follows:

(1) $Y \cup Z = \{x_i/(t_Y(x_i) \vee t_Z(x_i), 1 - f_Y(x_i \wedge f_Z(x_i))) | x_i \in X\}$;
(2) $Y \cap Z = \{x_i/(t_Y(x_i) \wedge t_Z(x_i), 1 - f_Y(x_i \vee f_Z(x_i))) | x_i \in X\}$;
(3) $Y \subseteq Z \Leftrightarrow t_Y(x_i) \leq t_Z(x_i)$ and $f_Y(x_i) \geq f_Z(x_i)$ for all $x_i \in X$;
(4) $Y^c = \{x_i/(f_Y(x_i), 1 - t_Y(x_i)) | x_i \in X\}$.

In the following, we introduce some basic concepts of lattice. Some axioms on an algebra $\mathbb{Q} = (X, \vee, \wedge)$ are given as follows:

(1) $x \wedge x = x, x \vee x = x$;
(2) $x \wedge y = y \wedge x, x \vee y = y \vee x$;
(3) $(x \wedge y) \wedge z = x \wedge (y \wedge z), (x \vee y) \vee z = x \vee (y \vee z)$;
(4) $(x \wedge y) \vee x = x, (x \vee y) \wedge x = x$;
(5) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z), x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$;
(6) $x_{max} = \max\{x_i | x_i \in X\}, x_{min} = \min\{x_i | x_i \in X\}$,
where $x, y, z \in X$.

If the algebra $\mathbb{Q}$ satisfies the axioms (1), (2) and (3), then it is called a quasilattice. If a quasilattice further satisfies the axiom (4), then it is called a lattice. If a lattice satisfies the axiom (5), then it is called a distributive lattice. If a lattice satisfies the axiom (6), then it is called a bounded lattice.

RESULTS

Multi-vague set

In this section, we propose a new concept called multi-vague set and define some basic operations and relations on multi-vague set. Some further results on multi-vague set are also investigated.

Definition 7. Let n be a positive integer. A multi-vague set $\tilde{V}$ in X is a set of ordered sequences
$\tilde{V} = \{x/(\mu_1(x), \mu_2(x), \cdots, \mu_i(x), \cdots, \mu_n(x)): x \in X\}$
$= \{x/([t_{\tilde{V}}^{(1)}(x), 1 - f_{\tilde{V}}^{(1)}(x)], [t_{\tilde{V}}^{(2)}(x), 1 - f_{\tilde{V}}^{(2)}(x)], \cdots, [t_{\tilde{V}}^{(n)}(x), 1 - f_{\tilde{V}}^{(n)}(x)]): x \in X\}$,
here $\mu_i \in \mathcal{V}(X), i = 1,2, \cdots, n$.

The function $\mu_{\tilde{V}} = (\mu_1, \mu_2, \cdots, \mu_n)$ is called the vague multi-membership function of multi-vague set $\tilde{V}$, n is called the dimension of $\tilde{V}$. The set of all multi-vague sets of dimension n in X is denoted by $\mathcal{MV}^n(X)$.

Definition 8. Let $\tilde{V}$ be a multi-vague set of dimension n in X.

(1) If $\tilde{V} = \{x/((0,0), \cdots, (0,0)): x \in X\}$, then $\tilde{V}$ is called the null multi-vague set of dimension n, denoted by $\overline{0}_n$.
(2) If $\tilde{V} = \{x/((1,1), \cdots, (1,1)): x \in X\}$, then $\tilde{V}$ is called the full multi-vague set of dimension n, denoted by $\overline{1}_n$.

In the following, we further define some basic operations and relations on multi-vague set.

Definition 9. Let $\tilde{Y} = \{x/((t_{\tilde{Y}}^{(1)}(x), 1 - f_{\tilde{Y}}^{(1)}(x)), (t_{\tilde{Y}}^{(2)}(x), 1 - f_{\tilde{Y}}^{(2)}(x)), \cdots, (t_{\tilde{Y}}^{(n)}(x), 1 - f_{\tilde{Y}}^{(n)}(x))): x \in X\}$ and $\tilde{Z} = \{x/((t_{\tilde{Z}}^{(1)}(x), 1 - f_{\tilde{Z}}^{(1)}(x)), (t_{\tilde{Z}}^{(2)}(x), 1 - f_{\tilde{Z}}^{(2)}(x)), \cdots, (t_{\tilde{Z}}^{(n)}(x), 1 - f_{\tilde{Z}}^{(n)}(x))): x \in X\}$ be two multi-vague sets of dimension n in X.
(1) $\tilde{Y} \cup \tilde{Z} = \{x/((t_{\tilde{Y}}^{(1)}(x) \vee t_{\tilde{Z}}^{(1)}(x), 1 - f_{\tilde{Y}}^{(1)}(x) \wedge f_{\tilde{Z}}^{(1)}(x)), \cdots, (t_{\tilde{Y}}^{(n)}(x) \vee t_{\tilde{Z}}^{(n)}(x), 1 - f_{\tilde{Y}}^{(n)}(x) \wedge f_{\tilde{Z}}^{(n)}(x))): x \in X\}$.
(2) $\tilde{Y} \cap \tilde{Z} = \{x/((t_{\tilde{Y}}^{(1)}(x) \wedge t_{\tilde{Z}}^{(1)}(x), 1 - f_{\tilde{Y}}^{(1)}(x) \vee f_{\tilde{Z}}^{(1)}(x)), \cdots, (t_{\tilde{Y}}^{(n)}(x) \wedge t_{\tilde{Z}}^{(n)}(x), 1 - f_{\tilde{Y}}^{(n)}(x) \vee f_{\tilde{Z}}^{(n)}(x))): x \in X\}$.
(3) $\tilde{Y}^c = \{x/((f_{\tilde{Y}}^{(1)}(x), 1 - t_{\tilde{Y}}^{(1)}(x)), \cdots, (f_{\tilde{Y}}^{(n)}(x), 1 - t_{\tilde{Y}}^{(n)}(x))): x \in X\}$.
(4) $\tilde{Y} \subseteq \tilde{Z} \Leftrightarrow$ for all $x \in X, t_{\tilde{Y}}^{(i)}(x) \leq t_{\tilde{Z}}^{(i)}(x)$ and $f_{\tilde{Y}}^{(i)}(x) \leq f_{\tilde{Y}}^{(i)}(x)$, $1 \leq i \leq n$.
(5) $\tilde{Y} = \tilde{Z} \Leftrightarrow \tilde{Y} \subseteq \tilde{Z}$ and $\tilde{Z} \subseteq \tilde{Y}$.

Theorem1 Let $\tilde{R} = \{x/((t_{\tilde{R}}^{(1)}(x), 1 - f_{\tilde{R}}^{(1)}(x)), (t_{\tilde{R}}^{(2)}(x), 1 - f_{\tilde{R}}^{(2)}(x)), \cdots, (t_{\tilde{R}}^{(n)}(x), 1 - f_{\tilde{R}}^{(n)}(x))): x \in X\}$, $\tilde{S} = \{x/((t_{\tilde{S}}^{(1)}(x), 1 - f_{\tilde{S}}^{(1)}(x)), (t_{\tilde{S}}^{(2)}(x), 1 - f_{\tilde{S}}^{(2)}(x)), \cdots, (t_{\tilde{S}}^{(n)}(x), 1 - f_{\tilde{S}}^{(n)}(x))): x \in X\}$ and $\tilde{Z} = \{x/((t_{\tilde{T}}^{(1)}(x), 1 - f_{\tilde{T}}^{(1)}(x)), (t_{\tilde{T}}^{(2)}(x), 1 - f_{\tilde{T}}^{(2)}(x)), \cdots, (t_{\tilde{T}}^{(n)}(x), 1 - f_{\tilde{T}}^{(n)}(x))): x \in X\}$ be three multi-vague sets of dimension n in X. Then

(1) $(\tilde{R} \cap \tilde{S}) \cap \tilde{T} = \tilde{R} \cap (\tilde{S} \cap \tilde{T})$;
(2) $\tilde{R} \cap (\tilde{S} \cup \tilde{T}) = (\tilde{R} \cap \tilde{S}) \cup (\tilde{R} \cap \tilde{T})$;
(3) $(\tilde{R} \cup \tilde{S}) \cup \tilde{T} = \tilde{R} \cup (\tilde{S} \cup \tilde{T})$;
(4) $\tilde{R} \cup (\tilde{S} \cap \tilde{T}) = (\tilde{R} \cup \tilde{S}) \cap (\tilde{R} \cup \tilde{T})$.

Proof. we only prove (1) and (2) since (3) and (4) can be proved similarly.

(1) Since $\tilde{Y} \cup \tilde{Z} = \{x/((t_{\tilde{Y}}^{(1)}(x) \vee t_{\tilde{Z}}^{(1)}(x), 1 - f_{\tilde{Y}}^{(1)}(x) \wedge t_{\tilde{Z}}^{(1)}(x)), \cdots, (t_{\tilde{Y}}^{(n)}(x) \vee t_{\tilde{Z}}^{(n)}(x), 1 - f_{\tilde{Y}}^{(n)}(x) \wedge t_{\tilde{Z}}^{(n)}(x))): x \in X\}$, then $(\tilde{Y} \cup \tilde{Z})^c = \{x/((f_{\tilde{Y}}^{(1)}(x) \wedge f_{\tilde{Z}}^{(1)}(x), 1 - t_{\tilde{Y}}^{(1)}(x) \vee t_{\tilde{Z}}^{(1)}(x)), \cdots, (f_{\tilde{Y}}^{(n)}(x) \wedge f_{\tilde{Z}}^{(n)}(x), 1 - t_{\tilde{Y}}^{(n)}(x) \vee t_{\tilde{Z}}^{(n)}(x))): x \in X\} = \tilde{Y}^c \cap \tilde{Z}^c$.
(2) $(\tilde{Y} \cup \tilde{Z}) \cap \tilde{Y} = \{x/(((t_{\tilde{Y}}^{(1)}(x) \vee t_{\tilde{Z}}^{(1)}(x)) \wedge t_{\tilde{Y}}^{(1)}(x), 1 - (f_{\tilde{Y}}^{(1)}(x) \wedge f_{\tilde{Z}}^{(1)}(x)) \vee f_{\tilde{Y}}^{(1)}(x)), \cdots, ((t_{\tilde{Y}}^{(n)}(x) \vee t_{\tilde{Z}}^{(n)}(x)) \wedge t_{\tilde{Y}}^{(n)}(x), 1 - (f_{\tilde{Y}}^{(n)}(x) \wedge f_{\tilde{Z}}^{(n)}(x)) \vee f_{\tilde{Y}}^{(n)}(x))): x \in X\}$
$= \{x/(((t_{\tilde{Y}}^{(1)}(x), 1 - f_{\tilde{Y}}^{(1)}(x)), \cdots, (t_{\tilde{Y}}^{(n)}(x), 1 - f_{\tilde{Y}}^{(n)}(x))): x \in X\} = \tilde{Y}$.

Multi-Vague soft set

In this section, we introduce an extended soft set model which is called multi-vague soft set by combining the multi-vague set and soft set. Some operations and their properties on multi-vague soft set will also be discussed.

Definition 10. Let $A \subseteq Q$. A pair $(\tilde{F}, A)$ is called a multi-vague soft set of dimension n over X, where $\tilde{F}: A \rightarrow \mathcal{MV}^n(X)$ is a mapping. In other words, a multi-vague soft set of dimension n over X is a parameterized family of multi-vague set of the universe X. To illustrate this idea, let us consider the following example (24).

Example 1. Let $X = \{u_1, u_2, u_3, u_4\}$ be the set of color cloths under consideration and $A = \{q_1, q_2, q_3\} \subseteq E$ be the set of parameters, where q_1 stands for 'color' which consists of red, green and blue, q_2 stands for 'ingredient' which is made from wool, cotton and acrylic, and q_3 stands for 'price' which can be various: high, medium and low. Let $\tilde{F}: A \rightarrow \mathcal{MV}^n(X)$ be a function given as follows:

$\tilde{F}(q_1) = \{u_1/((0.3,0.5), (0.2,0.6), (0.7,0.9)), u_2/((0.4,0.6), (0.3,0.7), (0.8,1)),$
$u_3/((0.6,0.9), (0.3,0.7), (0.5,0.7)), u_4/((0.8,0.9), (0.6,0.8), (0.2,0.5))\}$,
$\tilde{F}(q_2) = \{u_1/((0.4,0.7), (0.7,0.8), (0.3,0.5)), u_2/((0.6,0.8), (0.3,0.4), (0.7,0.9)),$
$u_3/((0.4,0.5), (0.7,0.8), (0.6,0.9)), u_4/((0.2,0.4), (0.6,0.9), (0.5,0.7))\}$,
$\tilde{F}(q_3) = \{u_1/((0.2,0.5), (0.4,0.6), (0.3,0.7)), u_2/((0.8,0.9), (0,5,0.7), (0.4,0.6)),$
$u_3/((0.5,0.8), (0.4,0.5), (0.7,0.9)), u_4/((0.3,0.6), (0.5,0.7), (0.8,0.9))\}$.

Then $(\tilde{F}, A)$ is a multi-vague soft set of dimension 3.

Remark 1. (1) If A has only an element, i.e. $A = \{q\}$, then multi-vague soft set becomes multi-vague set introduced in Section 3;
(2) If $t_i(u) = 1 - f_i(u), 1 \leq i \leq n$ for all $q \in A$ and $x \in X$, then multi-vague soft set degenerates to multi-fuzzy soft set (16);
(3) If $\tilde{F}(q)$ is a multi-vague set of dimension 1 for each $q \in A$, then multi-vague soft set reduces to vague soft set (18).

Definition 11. Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two multi-vague soft sets of dimension n over X. Then $(\tilde{F},A)$ is called a multi-vague soft subset of $(\tilde{G},B)$ if

(1) $A \subseteq B$;
(2) $\tilde{F}(q) \subseteq \tilde{G}(q)$ for all $q \in A$.

In this case, the above relationship is denoted by $(\tilde{F},A) \tilde{\subseteq} (\tilde{G},B)$. And $(\tilde{G},B)$ is said to be a multi-vague soft superset of $(\tilde{F},A)$.

Definition 12. Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two multi-vague soft sets of dimension n over X. Then $(\tilde{F},A)$ and $(\tilde{G},B)$ are said to be multi-vague soft equal if and only if $(\tilde{F},A) \tilde{\subseteq} (\tilde{G},B)$ and $(\tilde{G},B) \tilde{\subseteq} (\tilde{F},A)$.

Definition 13. The complement of a multi-vague soft set $(\tilde{F},A)$ of dimension n over X is denoted by $(\tilde{F},A)^c$ and is defined by $(\tilde{F},A)^c = (\tilde{F}^c,A)$, where $\tilde{F}^c: A \to \mathcal{MV}^n(X)$ is a mapping given by $\tilde{F}^c(q) = (\tilde{F}(q))^c$.

Example 2. We consider the multi-vague soft set $(\tilde{F},A)$ given in Example , and define another multi-vague soft set $(\tilde{G},B)$ of dimension n over X as follows:

$$\tilde{G}(q_1) = \{u_1/((0.2,0.3),(0.1,0.4),(0.3,0.7)), u_2/((0.2,0.5),(0.1,0.3),(0.6,0.8)),$$
$$u_3/((0.3,0.5),(0.2,0.4),(0.4,0.5)), u_4/((0.6,0.8),(0.3,0.4),(0.1,0.3))\},$$
$$\tilde{G}(q_2) = \{u_1/((0.3,0.4),(0.4,0.6),(0.2,0.3)), u_2/((0.4,0.5),(0.1,0.2),(0.5,0.8)),$$
$$u_3/((0.2,0.4),(0.5,0.7),(0.4,0.6)), u_4/((0.1,0.3),(0.4,0.7),(0.3,0.6))\}.$$

Then the $(\tilde{G},B)$ is a multi-vague soft subset of $(\tilde{F},A)$, and the complement of the $(\tilde{G},B)$ is

$$\tilde{G}^c(q_1) = \{u_1/((0.7,0.8),(0.6,0.9),(0.3,0.7)), u_2/((0.5,0.8),(0.7,0.9),(0.2,0.4)),$$
$$u_3/((0.5,0.7),(0.6,0.8),(0.5,0.6)), u_4/((0.2,0.4),(0.6,0.7),(0.7,0.9))\},$$
$$\tilde{G}^c(q_2) = \{u_1/((0.6,0.7),(0.4,0.6),(0.7,0.8)), u_2/((0.5,0.6),(0.8,0.9),(0.2,0.5)),$$
$$u_3/((0.6,0.8),(0.3,0.5),(0.4,0.6)), u_4/((0.7,0.9),(0.3,0.6),(0.4,0.7))\},$$

Definition 14. Let $(\tilde{F},A)$ be a multi-vague soft set of dimension n over X. Then

(1) $(\tilde{F},A)$ is said to be an empty multi-vague soft set, denoted by $\tilde{\Phi}_A^n$, if $F(e) = \overline{0}_n$ for each $q \in A$;
(2) $(\tilde{F},A)$ is said to be a full multi-vague soft set, denoted by $\tilde{\mathcal{I}}_A^n$, if $F(e) = \overline{1}_n$ for each $q \in A$.

Based on the above definitions, we can obtain easily the following properties.

Theorem 3. Let (F,A), (G,B) and (H,C) be three multi-vague soft sets of dimension n over X. Then

(1) $(\tilde{\Phi}_A^n)^c = \tilde{\mathcal{I}}_A^n$;
(2) $(\tilde{\mathcal{I}}_A^n)^c = \tilde{\Phi}_A^n$;
(3) $((\tilde{F},A)^c)^c = (\tilde{F},A)$;
(4) $(\tilde{\Phi}_A^n) \tilde{\subseteq} (\tilde{F},A)$;
(5) $(\tilde{F},A) \tilde{\subseteq} \tilde{\mathcal{I}}_A^n$;
(6) $(\tilde{F},A) \tilde{\subseteq} (G,B) \tilde{\subseteq} (H,C) \Longrightarrow (\tilde{F},A) \tilde{\subseteq} (H,C)$.

Proof. The proofs can be easily obtained from Definitions 11, 12 and 14.

Definition 15. Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two multi-vague soft sets of dimension n over X and $A,B \subseteq Q$. We define a mapping $\widetilde{H}: A \cup B \to \mathcal{MV}^n(X)$ such that for all $q \in A \cup B \neq \emptyset$,

$$\widetilde{H}(q) = \begin{cases} \tilde{F}(q), & \text{if } q \in A - B, \\ \tilde{G}(q), & \text{if } q \in B - A, \\ \widetilde{H}(q), & \text{if } q \in A \cap B. \end{cases}$$

(1) If $\widetilde{H}(q) = \tilde{F}(q) \cup \tilde{G}(q)$, then $(\widetilde{H}, A \cup B)$ is called the extended union of $(\tilde{F},A)$ and $(\tilde{G},B)$, denoted by $(\tilde{F},A) \tilde{\cup} (\tilde{G},B)$.
(2) If $\widetilde{H}(q) = \tilde{F}(q) \cap \tilde{G}(q)$, then $(\widetilde{H}, A \cup B)$ is called the extended intersection of $(\tilde{F},A)$ and $(\tilde{G},B)$, denoted by $(\tilde{F},A) \tilde{\cap} (\tilde{G},B)$.
If $A \cup B = \emptyset$, then $(\tilde{F},A) \tilde{\cup} (\tilde{G},B) = \tilde{\Phi}_\emptyset^n$ and $(\tilde{F},A) \tilde{\cap} (\tilde{G},B) = \tilde{\Phi}_\emptyset^n$.

Definition 16. Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be multi-vague soft sets of dimension n over X and $A,B \subseteq Q$. We define a mapping $\widetilde{H}: A \cap B \to \mathcal{MV}^n(X)$ such that for all $q \in A \cap B \neq \emptyset$,

(1) If $\widetilde{H}(q) = \tilde{F}(q) \cup \tilde{G}(q)$, then $(\widetilde{H}, A \cap B)$ is called the strict union of $(\tilde{F},A)$ and $(\tilde{G},B)$, denoted by $(\tilde{F},A) \Cup (\tilde{G},B)$.
(2) If $\widetilde{H}(q) = \tilde{F}(q) \cap \tilde{G}(q)$, then $(\widetilde{H}, A \cup B)$ is called the strict intersection of $(\tilde{F},A)$ and $(\tilde{G},B)$, denoted by $(\tilde{F},A) \Cap (\tilde{G},B)$.

Example 3. We consider the multi-vague soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ given in Example 1 and Example 2 respectively. Then

$((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))(q_1) = ((\tilde{F}, A) \Cap (\tilde{G}, B))(q_1)$
$= \{u_1/((0.2,0.3), (0.1,0.4), (0.3,0.7)), u_2/((0.2,0.5), (0.1,0.3), (0.6,0.8)),$
$u_3/((0.3,0.5), (0.2,0.4), (0.4,0.5)), u_4/((0.6,0.8), (0.3,0.4), (0.1,0.3))\},$
$((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))(q_2) = ((\tilde{F}, A) \Cap (\tilde{G}, B))(q_2)$
$= \{u_1/((0.3,0.4), (0.4,0.6), (0.2,0.3)), u_2/((0.4,0.5), (0.1,0.2), (0.5,0.8)),$
$u_3/((0.2,0.4), (0.5,0.7), (0.4,0.6)), u_4/((0.1,0.3), (0.4,0.7), (0.3,0.6))\},$
$((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B))(q_1) = ((\tilde{F}, A) \Cup (\tilde{G}, B))(q_1)$
$= \{u_1/((0.3,0.5), (0.2,0.6), (0.7,0.9)), u_2/((0.4,0.6), (0.3,0.7), (0.8,1)),$
$u_3/((0.6,0.9), (0.3,0.7), (0.5,0.7)), u_4/((0.8,0.9), (0.6,0.8), (0.2,0.5))\},$
$((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B))(q_2) = ((\tilde{F}, A)(q_2) \Cup (\tilde{G}, B))(q_2)$
$= \{u_1/((0.4,0.7), (0.7,0.8), (0.3,0.5)), u_2/((0.6,0.8), (0.3,0.4), (0.7,0.9)),$
$u_3/((0.4,0.5), (0.7,0.8), (0.6,0.9)), u_4/((0.2,0.4), (0.6,0.9), (0.5,0.7))\},$
$((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B))(q_3) = ((\tilde{F}, A)(q_3) \cap (\tilde{G}, B))(q_3) = \tilde{F}(q_3)$
$= \{u_1/((0.2,0.5), (0.4,0.6), (0.3,0.7)), u_2/((0.8,0.9), (0,5,0.7), (0.4,0.6)),$
$u_3/((0.5,0.8), (0.4,0.5), (0.7,0.9)), u_4/((0.3,0.6), (0.5,0.7), (0.8,0.9))\}.$

Theorem 4. Let $A \subseteq Q$, $(\tilde{F}, A)$ be a multi-vague soft set of dimension n over X. Then

(1) $(\tilde{F}, A) \Cap \tilde{\mathcal{J}}_E^n = (\tilde{F}, A)$;
(2) $(\tilde{F}, A) \tilde{\cap} \tilde{\mathcal{J}}_A^n = (\tilde{F}, A)$;
(3) $(\tilde{F}, A) \tilde{\cup} \tilde{\mathcal{J}}_A^n = \tilde{\mathcal{J}}_A^n$;
(4) $(\tilde{F}, A) \Cup \tilde{\mathcal{J}}_E^n = \tilde{\mathcal{J}}_A^n$;
(5) $(\tilde{F}, A) \Cap \tilde{\Phi}_E^n = \tilde{\Phi}_A^n$;
(6) $(\tilde{F}, A) \tilde{\cap} \tilde{\Phi}_A^n = \tilde{\Phi}_A^n$;
(7) $(\tilde{F}, A) \tilde{\cup} \tilde{\Phi}_A^n = (\tilde{F}, A)$;
(8) $(\tilde{F}, A) \Cup \tilde{\Phi}_E^n = (\tilde{F}, A)$;
(9) $(\tilde{F}, A) \Cap \tilde{\Phi}_\phi^n = \tilde{\Phi}_\phi^n$;
(10) $(\tilde{F}, A) \Cup \tilde{\Phi}_\phi^n = \tilde{\Phi}_\phi^n$;
(11) $(\tilde{F}, A) \tilde{\cap} \tilde{\Phi}_\phi^n = (\tilde{F}, A)$;
(12) $(\tilde{F}, A) \tilde{\cup} \tilde{\Phi}_\phi^n = (\tilde{F}, A)$.

Proof. It is easily obtained from Definitions 14,15 and 16.

Theorem 5. Let $A, B \subseteq Q$, $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two multi-vague soft sets of dimension n over X. Then

(1) $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))^c = (\tilde{F}, A)^c \tilde{\cup} (\tilde{G}, B)^c$;
(2) $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B))^c = (\tilde{F}, A)^c \tilde{\cap} (\tilde{G}, B)^c$,
(3) $((\tilde{F}, A) \Cap (\tilde{G}, B))^c = (\tilde{F}, A)^c \Cup (\tilde{G}, B)^c$;
(4) $((\tilde{F}, A) \Cup (\tilde{G}, B))^c = (\tilde{F}, A)^c \Cap (\tilde{G}, B)^c$.

Proof. We only prove (1). By using a similar technique, (2)-(4) can be proved, too.

Suppose that $(\tilde{F}, A) \tilde{\cap} (\tilde{G}, B) = (\tilde{H}, C)$. Then $C = A \cup B$,

(i) if $C = \phi$, then $A = \phi$ and $B = \phi$. Hence $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))^c = \tilde{\Phi}_\phi^n$=$(\tilde{F}, A)^c \tilde{\cap} (\tilde{G}, B)^c$.
(ii) if $C \neq \phi$, then for each $q \in C$ and $x \in X$, we have

$$\tilde{H}(q) = \begin{cases} \tilde{F}(q), & \text{if } q \in A - B, \\ \tilde{G}(q), & \text{if } q \in B - A, \\ \tilde{F}(q) \cap \tilde{G}(q), & \text{if } q \in A \cap B. \end{cases}$$

Then

$$\tilde{H}^c(q) = \begin{cases} \tilde{F}^c(q), & \text{if } q \in A - B, \\ \tilde{G}^c(q), & \text{if } q \in B - A, \\ (\tilde{F}(q) \cap \tilde{G}(q))^c, & \text{if } q \in A \cap B. \end{cases}$$

Again suppose that $(\tilde{F}, A)^c \cup (\tilde{G}, B)^c = (\tilde{J}, D)$. Then $D = A \cup B$ and for each $q \in D$, we have

$$\tilde{J}(q) = \begin{cases} \tilde{F}^c(q), & \text{if } q \in A - B, \\ \tilde{G}^c(q), & \text{if } q \in B - A, \\ \tilde{F}^c(q) \cup \tilde{G}^c(q), & \text{if } q \in A \cap B. \end{cases}$$

By (2) in Theorem , we have $\tilde{F}^c(q) \cup \tilde{G}^c(q) = (\tilde{F}(q) \cap \tilde{G}(q))^c$, i.e., $\tilde{J}(q) = \tilde{H}^c(q)$ for all $q \in A$. Therefore, $(\tilde{H}^c, C)$ and $(\tilde{J}, M)$ are the same multi-vague soft sets. It follows that $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))^c = (\tilde{F}, A)^c \tilde{\cup} (\tilde{G}, B)^c$.

Theorem 6. Let $\diamond \in \{\tilde{\cap}, \tilde{\Cap}, \tilde{\cup}, \tilde{\Cup}\}, A, B, C \subseteq Q$, $(\tilde{F}, A)$, $(\tilde{G}, B)$ and $(\tilde{H}, C)$ be three multi-vague soft sets of dimension n over X. Then

(1) $(\tilde{F}, A) \diamond (\tilde{F}, A) = (\tilde{F}, A)$;
(2) $(\tilde{F}, A) \diamond (\tilde{G}, B) = (\tilde{G}, B) \diamond (\tilde{F}, A)$;
(3) $(\tilde{F}, A) \diamond ((\tilde{G}, B) \diamond (\tilde{H}, C)) = ((\tilde{F}, A) \diamond (\tilde{G}, B)) \diamond (\tilde{H}, C)$.

Proof. (1) and (2) are trivial. We only prove (3). For example, let $\diamond = \tilde{\cap}$, the others can be proved similarly.

Suppose that $(\tilde{F}, A) \tilde{\cap} ((\tilde{G}, B) \tilde{\cap} (\tilde{H}, C)) = (\tilde{J}, M)$ and $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)) \tilde{\cap} (\tilde{H}, C) = (\tilde{K}, N)$, so $M = N = A \cup B \cup C$. If $M = \phi$, then it is clear that the equality holds. If $M \neq \phi$, then for all $q \in M$, it follows that $q \in A$, or $q \in B$, or $q \in C$. Without losing generality, we suppose that $q \in A$.

(i) If $q \notin B$ and $q \notin C$, then $\tilde{J}(q) = \tilde{F}(q) = \tilde{K}(q)$;
(ii) If $q \in B$ and $q \notin C$, then $\tilde{J}(q) = \tilde{F}(q) \cap \tilde{G}(q) = \tilde{K}(q)$;
(iii) If $q \notin B$ and $q \in C$, then $\tilde{J}(q) = \tilde{F}(q) \cap \tilde{H}(q) = \tilde{K}(q)$;
(iv) If $q \in B$ and $q \in C$, then $\tilde{J}(q) = (\tilde{F}(q) \cap \tilde{G}(q)) \cap \tilde{H}(q)$ and $\tilde{K}(q) = \tilde{F}(q) \cap (\tilde{G}(q) \cap \tilde{H}(q))$. By (1) in Theorem , we have $(\tilde{F}(q) \cap \tilde{G}(q)) \cap \tilde{H}(q) = \tilde{F}(q) \cap (\tilde{G}(q) \cap \tilde{H}(q))$. Hence, $\tilde{J}(q) = \tilde{K}(q)$.

To sum up, $\tilde{J}$ and $\tilde{K}$ are indeed the same set-valued mappings. Thus $(\tilde{J}, M) = (\tilde{K}, N)$, i.e. $(\tilde{F}, A) \tilde{\cap} ((\tilde{G}, B) \tilde{\cap} (\tilde{H}, C)) = ((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)) \tilde{\cap} (\tilde{H}, C)$.

Let £(X,Q) denote the set of all multi-vague soft sets of dimension n over X. Then based on Theorem 6, we have the following result.

Proposition 1. Let $\delta \in \{\tilde{\cap}, \tilde{\Cap}\}$ and $\theta \in \{\tilde{\cup}, \tilde{\Cup}\}$, then (£($X,Q$),$\delta$,$\theta$) is a quasilattice.

Theorem 7. Let $A, B \subseteq Q$, $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two multi-vague soft sets of dimension n over X. Then

(1) $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) \tilde{\Cap} (\tilde{F}, A) = (\tilde{F}, A)$;
(2) $((\tilde{F}, A) \tilde{\Cup} (\tilde{G}, B)) \tilde{\cap} (\tilde{F}, A) = (\tilde{F}, A)$;
(3) $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)) \tilde{\Cup} (\tilde{F}, A) = (\tilde{F}, A)$;
(4) $((\tilde{F}, A) \tilde{\Cap} (\tilde{G}, B)) \tilde{\cup} (\tilde{F}, A) = (\tilde{F}, A)$.
Proof. we only prove (1). (2),(3) and (4) can be proved in the similar way.

Let $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) = (\tilde{J}, M)$ and $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) \tilde{\Cap} (\tilde{F}, A) = (\tilde{K}, N)$. Then $M = A \cup B$, $N = (A \cup B) \cap A = A$. And for each $q \in A$ and $x \in X$,
(i) if $q \notin B$, then $\tilde{K}(q) = \tilde{J}(q) \cap \tilde{F}(q) = \tilde{F}(q) \cap \tilde{F}(q) = \tilde{F}(q)$.
(ii) if $q \in B$, then $\tilde{K}(q) = \tilde{J}(q) \cap \tilde{F}(q) = (\tilde{F}(q) \cup \tilde{G}(q)) \cap \tilde{F}(q)$. By (2) in Theorem 2, we have $(\tilde{F}(q) \cup \tilde{G}(q)) \cap \tilde{F}(q) = \tilde{F}(q)$. Therefore, $\tilde{K}(q) = \tilde{F}(q)$.

All in all, $(\tilde{K}, N) = (\tilde{F}, A)$, i.e. $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) \tilde{\Cap} (\tilde{F}, A) = (\tilde{F}, A)$.

By Proposition 1 and Theorem 7, we have the following property.

Proposition 2. (£(X,Q),$\tilde{\cap}$,$\tilde{\Cup}$) and (£(X,Q),$\tilde{\Cap}$,$\tilde{\cup}$) are two lattices.

Theorem 8. Let $(\tilde{F}, A)$, $(\tilde{G}, B)$ and $(\tilde{H}, C)$ be three multi-vague soft sets of dimension n over X. Then

(1) $((\tilde{F}, A) \tilde{\Cap} (\tilde{G}, B)) \tilde{\cup} (\tilde{H}, C) = ((\tilde{F}, A) \tilde{\Cap} (\tilde{G}, B)) \tilde{\cup} ((\tilde{F}, A) \tilde{\Cap} (\tilde{H}, C))$;
(2) $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) \tilde{\Cap} (\tilde{H}, C) = ((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) \tilde{\Cap} ((\tilde{F}, A) \tilde{\cup} (\tilde{H}, C))$;
(3) $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)) \tilde{\Cup} (\tilde{H}, C) = ((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B)) \tilde{\Cup} ((\tilde{F}, A) \tilde{\cap} (\tilde{H}, C))$;
(4) $((\tilde{F}, A) \tilde{\Cup} (\tilde{G}, B)) \tilde{\cap} (\tilde{H}, C) = ((\tilde{F}, A) \tilde{\Cup} (\tilde{G}, B)) \tilde{\cap} ((\tilde{F}, A) \tilde{\Cup} (\tilde{H}, C))$.

Proof. we only prove (1). (2), (3) and (4) can be proved in the similar way.

Let $((\tilde{F}, A) \tilde{\Cap} (\tilde{G}, B)) \tilde{\cup} (\tilde{H}, C) = (\tilde{J}, M)$ and $((\tilde{F}, A) \tilde{\Cap} (\tilde{G}, B)) \tilde{\cup} ((\tilde{F}, A) \tilde{\Cap} (\tilde{H}, C)) = (\tilde{K}, N)$. Then $M = A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = N$. For each $q \in M$, it follows that $q \in A$ and $q \in B \cup C$.
(i) If $q \in A, q \notin B, q \in C$, then $\tilde{J}(q) = \tilde{F}(q) \cap \tilde{H}(q) = \tilde{K}(q)$.
(ii) If $q \in A, q \in B, q \notin C$, then $\tilde{J}(q) = \tilde{F}(q) \cap \tilde{G}(q) = \tilde{K}(q)$.
(iii) If $q \in A, q \in B, q \in C$, then $\tilde{J}(q) = \tilde{F}(q) \cap (\tilde{G}(q) \cup \tilde{H}(q))$ and $(\tilde{F}(q) \cap \tilde{G}(q) \cup (\tilde{F}(q) \cap \tilde{H}_\gamma) = \tilde{K}(q)$. By (2) in Theorem 1, we have $\tilde{F}(q) \cap (\tilde{G}(q) \cup \tilde{H}(q)) = (\tilde{F}(q) \cap \tilde{G}(q) \cup (\tilde{F}(q) \cap \tilde{H}(q))$. Thus $\tilde{J}(q) = \tilde{K}(q)$.

Theorem 9. $(\pounds(X,Q),\widetilde{\cap},\widetilde{\Cup})$ and $(\pounds(X,Q),\widetilde{\Cap},\widetilde{\cup})$ are two bounded distributive lattices.

Proof. By Proposition 2 and Theorem 7, we get that $(\pounds(X,Q),\widetilde{\cap},\widetilde{\Cup})$ and $(\pounds(X,Q),\widetilde{\Cap},\widetilde{\cup})$ are two distributive lattices. It is clear that $\widetilde{\Phi}_Q^n$ and $\widetilde{\mathcal{J}}_Q^n$ are the minimum element and the maximum element both in $(\pounds(X,Q),\widetilde{\cap},\widetilde{\Cup})$ and $(\pounds(X,Q),\widetilde{\Cap},\widetilde{\cup})$, respectively

Conclusion

This work can be viewed as a continuation of the studies of Yang et al. (16) and Yang et al. (18). Here we have further extended multi-fuzzy set to multi-vague set and studied some properties of its basic operations. We also have proposed a new hybrid model called multi-vague soft set by combining the proposed multi-vague set and soft set models. It has been pointed out that multi-vague soft set is an extension of many existing soft set models, such as multi-vague set, multi-fuzzy soft set and vague soft set. Some operations on multi-vague soft set have been defined and some basic properties of those operations have been discussed. Finally, We hope that our work would be useful to handle some other realistic uncertain problems.

Acknowledgement: The authors would like to thank the associate editor and the anonymous referees for their insightful and constructive comments and suggestions. This work is supported by the National Natural Science Foundation of China (Nos. 61473118, 71501135), China Postdoctoral Science Foundation (Nos. 2015M572482), Natural Science Foundation of Hunan Province (No. 2019JJ40100), Project of improving the basic ability of Young and Middle-aged Teachers in Higher Educational Instituions of Guanxi Province (No.2018KY0563) and the Doctor's Scientific Research Foundation of Hezhou University (No. HZUBS201505).

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