IJIRR

International Journal of Information Research and Review
Vol. 06, Issue, 04, pp.6230-6234, April, 2019

RESEARCH ARTICLE

VIRTUAL EMPIRICAL RESEARCH AND UNDERSTANDING DOUBLE INTEGRAL

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ARTICLE INFO

Article History:
Received 14th January, 2019
Received in revised form
10th February, 2019
Accepted 20th March, 2019
Published online 30th April, 2019

Keywords:
APOS theory, Definite integral, Riemann sum, Volume, Double Integral, Physics and engineering

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ABSTRACT

Research in mathematics and science education reveals a disconnect for students as they attempt to apply their mathematical knowledge to science and engineering. This means that teaching of a mathematical concept is easy for a teacher but learning is difficult for his or her students. With this conclusion in mind, this paper investigates a particular calculus topic that is used frequently in science and engineering: the double integral.. The results of this study describe that certain conceptualizations of the double integral, including the volume under a surface and the values of an anti-derivative, are limited in their ability to help students make sense of contextualized integrals. These focus on understanding the concept of double integral with virtual empirical experiment and APOS theory.

INTRODUCTION

In late decades, more and more attention has been given to compiling a body of research regarding student understanding of mathematics at the undergraduate level. Already this research has provided much information about how students learn and understand a variety of concepts from calculus, differential equations, statistics, and mathematical proof. Among calculus concepts, researchers have focused heavily on student thinking about limits (e.g., Bezuidenhout, 2001; Davis and Vinner, 1986; Oehrtman, Carlson and Thompson, 2008; Oehrtman, 2004; Tall and Vinner, 1981; Williams, 1991), but have also provided insight about how students understand the derivative (e.g., Marrongelle, 2004; Orton, 1983b; Zandieh, 2000) and Riemann sums and the integral (e.g., Bezuidenhout and Olivier, 2000; Hall, 2010; Orton, 1983a; Rasslan and Tall, 2002; Sealey and Oehrtman, 2005; Sealey and Oehrtman, 2007; Sealey, 2006; Thompson and Silverman, 2008; Thompson, 1994). Overall, the concepts of the derivative and the integral are less explored than the idea of the limit. While the limit is fundamental to calculus, the derivative and the integral have additional layers of meaning above and beyond the limit, as well as meanings that do not necessarily require accessing the concept of a limit (Marrongelle, 2004; Sealey and Oehrtman, 2007; Thompson and Silverman, 2008; Zandieh, 2000). Thus, the derivative and the integral need special attention in order to learn how students understand the main ideas of first-year calculus. In particular, students' understanding of the integral is an especially valuable topic, since integration serves as the basis for many real world applications and subsequent coursework (Sealey and Oehrtman, 2005; Thompson and Silverman, 2008). The integral shows up in a variety of contexts within physics and engineering (Hibbeler, 2004, 2006; Serway and Jewett, 2008; Tipler and Mosca, 2008) and students who continue into further calculus courses will encounter the integral more often than the derivative (Salas, Hille and Etgen, 2006; Stewart, 2007; Thomas, Weir and Hass, 2009). However, an overreliance on certain interpretations of the integral, such as an "area under a curve," can limit the integral's applicability to these other areas (Sealey, 2006). Evidence of student difficulties with the integral has been documented over the years in several studies (Bezuidenhout and Olivier, 2000; Orton, 1983a; Rasslan and Tall, 2002; Tall, 1992; Thompson, 1994). Additionally, researchers have noted the perception among educators that students transitioning into science courses are routinely struggling to apply their mathematical knowledge to the science domain (Fuller, 2002; Gainsburg, 2006; Redish, 2005). This should be of primary concern for instructors of first-year calculus due to its nature as a service course and the large portion of science students enrolled in these classes (Ellis, Williams, Sadid, Bosworth and Stout, 2004; Ferrini-Mundy and Graham, 1991). Hall (2010) demonstrated several ways that students may interpret the definite and indefinite integral, including "area," "Riemann sums," "evaluation," and "language." Conceptions of the integral as an area or as a calculation appeared predominant among his students. Hall's main focus, however, was on the influence of informal language on students' thinking about the integral and he did not attempt to analyze the composition of their concept images (see Tall and Vinner, 1981). Sealey and others, on the other hand, primarily emphasized students' conceptualization of the integral as a Riemann sum (Engelke and Sealey, 2009; Sealey and Oehrtman, 2005, 2007; Sealey, 2006). These studies focus on how students connected the Riemann sum to concepts like the limit and how students used it in solving certain problems, such as approximating the force on a dam. Much of the work was centered on how ideas of accumulation and error were entwined with the conception of the Riemann sum. Thurston (1990) had recognized that, due in part to this possibility of synthesis, mathematics was tremendously compressible. He had also noted, that while the insight that went with this impression was one of the real joys of mathematics, this process was irreversible; therefore, it was very hard for the mathematician to put himself in the frame of mind of the student who had not yet achieved this synthesis. Discovering or rather rediscovering relationships is often considered among the most effective ways for children to learn mathematics. To some extent, this effectiveness may be attributed to the psychological aspects of the process of discovery: the personal involvement, the intensity of the attention, the feeling of achievement and success. Learning by discovery, however, is time-consuming, and this is one reason why teachers, especially teachers of more advanced mathematics, tend not to use it. The purpose of this paper is to describe a way for discovering or rediscovering by using computer activities.

Student participants and data collection: To capture data of students discussing the integral, interviews were conducted with eight participants. All eight students were interviewed in pairs, so that they could discuss their thinking with each other. The students were encouraged to verbally discuss their thinking with each other while they worked. Their written and spoken activities were videotaped and the researcher took notes. These comprise the primary sources of data for the study. The students chosen for this study had two important characteristics. First, since a large portion of first-year calculus classes is made up of science and engineering students, it was natural to include students from these disciplines. Second, participants were desired to be experienced calculus students, so that any lack of productiveness of the three conceptualizations under consideration could not simply be attributed to a lack of developed mathematical knowledge. In order to recruit participants who met these two characteristics, students were all recruited toward the end of a calculus-based physics course at their university. Students were only invited to participate who had completed the first calculus course at their university and had either completed, or nearly completed, the second calculus course. Students were only selected who had received a grade of A or B in these courses, or had a score of five on the relevant exam. As a result, all of the students in this study had experience working with definite integrals in both mathematics and physics contexts. The pseudonyms given to the students are to help suggest the pairs they worked in: Jusong, Gwangjin, Jina, Byol, MIrae, Miyon, Unyong, Bokgyong.

Virtual Empirical experiment-computer activity: As earlier stated, student's learning process(especially synthesis process) is irreversible, therefore, it is hard for teacher to put himself in the frame of mind of the student who has not yet achieved this synthesis. In general, to identify the mathematical concepts for the students, it is necessary for them to have the synthesis process and it requires a lot of time. This means that teaching of the teacher is easy but learning of the students is very hard. A good way for solving this problem is discovering or rediscovering, but it is time-consuming. However, by using virtual empirical experiment- computer activity, we can solve this problem. The first step in this approach is to make an initial theoretical analysis using our theoretical perspective on learning theory, the epistemology of the concept being studied based upon past research, literature, and the mathematical knowledge of the researchers. The purpose of the theoretical analysis is to propose a genetic decomposition or model of cognition: that is, a description of specific mental constructions that a learner might make in order to develop her or his understanding of the concept. These mental constructions are called actions, processes, objects, and schemas, so that the theoretical framework we use is sometimes referred to as the APOS Theory. According to APOS theory, an action is a transformation of mathematical objects that is performed by an individual according to some explicit algorithm and hence is seen by the subject as externally driven. When the individual reflects on the action and constructs an internal operation that performs the same transformation then we say that the action has been interiorized to a process. When it becomes necessary to perform actions on a process, the subject must encapsulate it to become a total entity, or an object. In many mathematical operations, it is necessary to de-encapsulate an object and work with the process from which it came. A schema is a coherent collection of processes, objects and previously constructed schemas, that is invoked to deal with a mathematical problem situation. As with encapsulated processes, an object is created when a schema is done to become another kind of object which can also be done to obtain the original contents of the schema.

Let's consider virtual empirical experiment-computer activities in detail. We provide the students with a set of instructions for using this activity, along with a series of specific questions to investigate. For a surface (defined $z = f(x, y)$) they are asked to partition a solid surround by the surface by themselves. Every student changes the value of N and investigate how the shape of the solid changes. Throughout these activities they can have the intuitional understanding as N is increased. (see Figure 1, 2, 3)

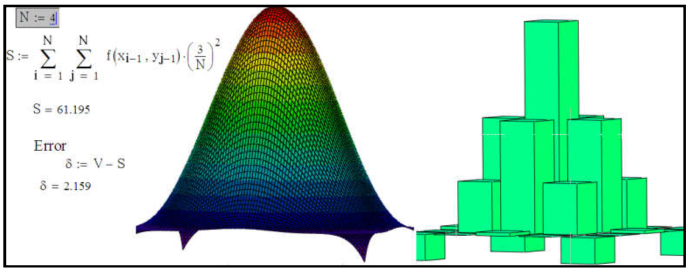

Figure 1. Case N=4

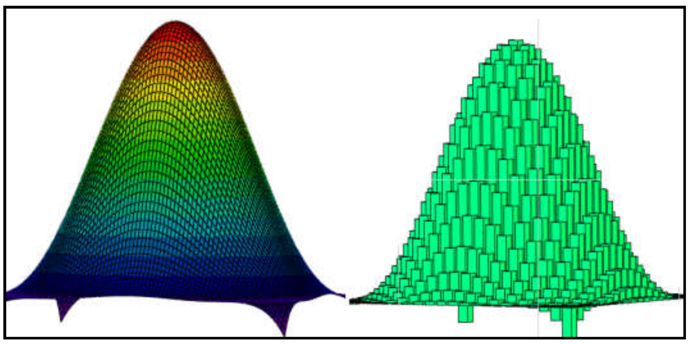

Figure 2. Case N=25

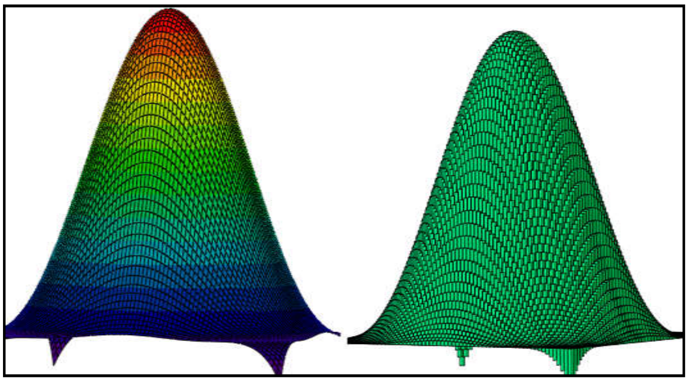

Figure 3. Case N=65

Gwangjin: Case N=4, the N-solid is very different from the solid.(N-solid : the solid consisting of N hexahedrons)

Jusong: but case N=25, the N-solid is similar to the solid. This means that the N-solid is more and more approximated to the solid as the number of the partition is increased.

Byol: right. By the way, another choice of the evaluation point brings the changes of the shape of the N-solid.
…
Bokgyong: If I have another choice of the evaluation point case N=3, the shape of the N-solid is very different from the original one. How do you think about this?
Jina: um…

Jina: But if the number of the partition is 6, the new N-solid is slightly similar to the original one.
Bokgyong: yeah,[quietly] .by the way…

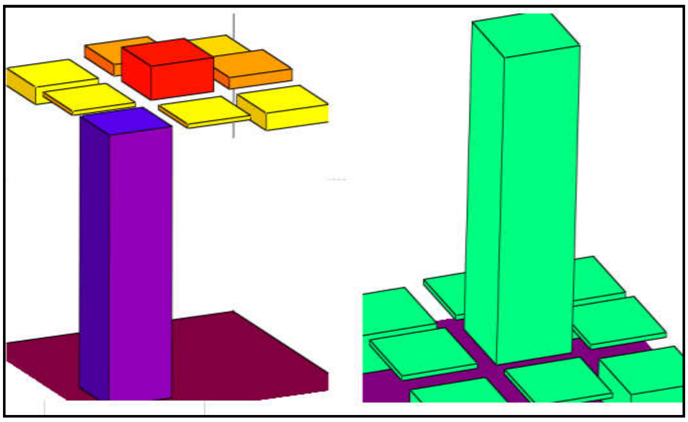

Figure 4.

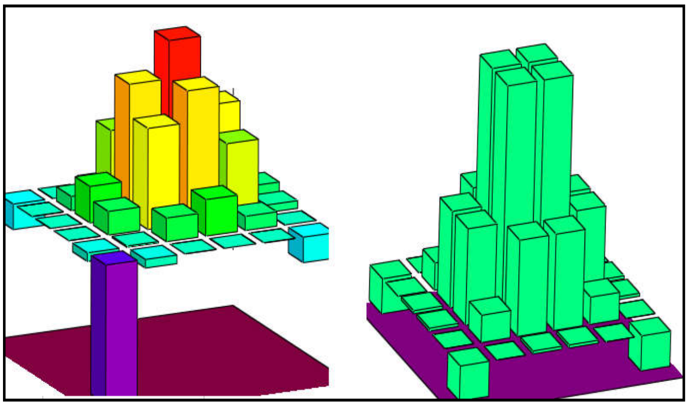

Figure 5.

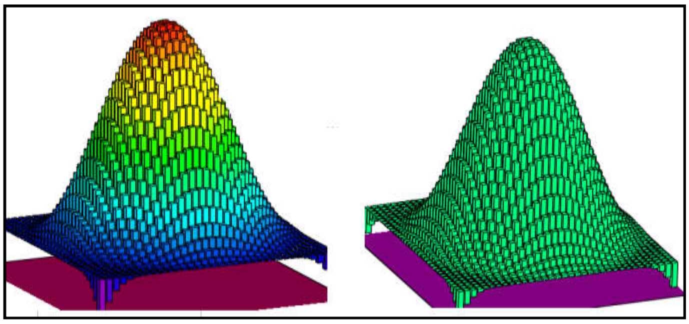

Figure 6.

DISCUSSION

Are virtual empirical experiment-computer activity effective? The reader may wish to know what evidence we are able to provide in support of the use of these computer activities. Indeed, what evidence is there that students are any more likely to achieve significant conceptual understanding using computer activities than they would without them? At this point, our evidence is anecdotal and judgmental. We have observed repeatedly that class time spent working with computer activities results in a higher level of engagement and interest than time spent in traditional lecture or class discussion. In a recent calculus class, for example, quite a few of the students were reluctant to leave at the end of the period because they were so involved with the computer activity. We have never experienced that sort of engagement in a traditional classroom activity. So, one point that can be made in favor of these activities is that students appear to value them and consider them interesting and worthwhile. But are the activities educationally worthwhile? In addition to finding the activities interesting and captivating, are the students actually learning anything? This is difficult to determine. In fact, it may be nearly impossible to quantify the precise role of any experience or set of experiences in a student's construction of knowledge. Even the students themselves are unlikely to recognize how a computer activity contributed to their ultimate understanding of a concept. Once they have acquired conceptual understanding, they may easily discount either its conceptual depth or the difficulty involved in achieving understanding.(This point is dramatically illustrated by the anecdote presented earlier of Hare's irate calculus student, who inferred from the simplicity of his insight that it had never been pointed out during previous instruction.) And it would obviously be silly to claim that conceptual understanding can be achieved only through the use of computer activities. These observations suggest that some sort of empirical demonstration of the impact of virtual empirical investigation would be very difficult to arrange.

Conclusion

Virtual empirical experiment helps the students their understanding mathematical concepts. In particular students themselves can find the idea of a mathematical concepts or a principle and understand the ones throughout the virtual empirical experiment-computer activities.

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