IJIRR

International Journal of Information Research and Review
Vol. 1, Issue, 10, pp.099-101, October, 2014

Full Length Research Article

ON THE TERNARY CUBIC EQUATION $5(X^2+Y^2)-9XY+X+Y+1=23Z^3$

***Vidhyalakshmi, S., Kavitha, A. and Gopalan, M. A.**

Department of Mathematics, Shrimati Indira Gandhi College, Trichy, India

ARTICLE INFO

Article History:
Received 07th July, 2014
Received in revised form
22nd August, 2014
Accepted 16th September, 2014
Published online 30th October, 2014

Keywords:

Ternary Cubic,
Integral Solutions.

ABSTRACT

The ternary cubic Diophantine equation is analyzed for its infinitely many non-zero distinct integral solutions. A few interesting properties among the solutions are presented.

INTRODUCTION

Integral solutions for the cubic homogeneous or non-homogeneous Diophantine equations are an interesting concept, as it can be seen from (Carmichael, 1959; Dickson, 2005 and Modrell, 1969). In (Gopalan, 2006,2007, 2008a,b,c,d, 2010 a,b,c,d,e,f,g, 2013a) a few special cases of ternary cubic Diophantine equations are studied. In this communication, we present the integral solutions of yet another ternary cubic equation $5(X^2+Y^2)-9XY+X+Y+1=23Z^3$. A few interesting relations between the solutions are obtained.

MATERIALS AND METHODS

The Diophantine equation to be solved for its non-zero distinct integral solution is,

$$5(X^2+Y^2)-9XY+X+Y+1=23Z^3 \quad \text{........(1)}$$

The substitution of linear transformations,

$$X=u+v, Y=u-v, u\neq v\neq 0 \quad \text{......(2)}$$

in (1) we get,

$$5((u+v)^2+(u-v)^2)-9(u+v)(u-v)+u+v+u-v+1=23Z^3$$

***Corresponding author: Vidhyalakshmi, S.**
Department of Mathematics, Shrimati Indira Gandhi College, Trichy, India.

$$u^2+2u+1+19v^2=23Z^3 \quad \text{...............(3)}$$

$$(u+1)^2+19v^2=23Z^3 \quad \text{............(4)}$$

Let $Z=a^2+19b^2$

where a,b are non-zero distinct integers. Different patterns of (1) are illustrated below.

PATTERN: 1

Write 23 as, $(2+i\sqrt{19})(2-i\sqrt{19})$ ....(5)

Substituting (4) and (5) in (3)

$$(u+1)^2+19v^2=(2+i\sqrt{19})(2-i\sqrt{19})(a+i\sqrt{19}b)^3(a-i\sqrt{19}b)^3$$

Employing positive and negative factors, we get,

$$(u+1+i\sqrt{19}v)=(2+i\sqrt{19})(a+i\sqrt{19}b)^3 \quad \text{...............(6)}$$

$$(u+1-i\sqrt{19}v)=(2-i\sqrt{19})(a-i\sqrt{19}b)^3 \quad \text{.........(7)}$$

Equating real and imaginary parts in (6)

$$u=2a^3+361b^3-114ab^2-57a^2b-1$$

$v=a^3-38b^3-57ab^2+6a^2b$

Hence the values of XandY satisfies (1) are given by,

$X(a,b)=3a^3+323b^3-171ab^2-51a^2b-1$ ....(8)

$Y(a,b)=a^3+399b^3-57ab^2-63a^2b-1$ ......(9)

Z(a,b)=a^2+19b^2

Thus (8), (9), and (4) represent non-zero distinct integral solutions of (1) in two parameters.

PROPERTIES

- $2cp_{9,a}-X(a,1)-T_{104,a}\equiv 0(Mod\,2)$
- $3Y(a,2)-X(a,2)+T_{554,a}\equiv 1(Mod\,5)$
- $Y(a,1)-cp_{6,a}+T_{128,a}\equiv 3(Mod\,5)$
- $18p_a^3-X(a,1)-T_{122,a}\equiv 0(Mod\,2)$
- $6\{Z(a,a(a+1))-76T_{3,a}^2\}$ is a nasty number

PATTERN:2

Assume $Z=a^2+19b^2$

Write 1 as, $1=\frac{(9+i\sqrt{19})(9-i\sqrt{19})}{100}$ ....(10)

(3) is written as $(u+1)^2+19v^2=23Z^3*1$ ....(11)

Substuting (4) and (10) in (11) and applying the method of factorization, wehave

$$(u+1+i\sqrt{19}v)(u+1-i\sqrt{19}v)=(2+i\sqrt{19})(2-i\sqrt{19})(a+i\sqrt{19}b)^3(a-i\sqrt{19}b)^3\frac{(9+i\sqrt{19})(9-i\sqrt{19})}{100}$$

Define,

$$(u+1+i\sqrt{19}v)=\frac{1}{10}(2+i\sqrt{19})(9+i\sqrt{19})(a+i\sqrt{19}b)^3$$

Equating real and imaginary parts, weget

$$u=\frac{1}{10}[-a^3+3971b^3+57ab^2-627a^2b-10]$$
$$v=\frac{1}{10}[11a^3+19b^3-627ab^2-3a^2b]$$

Substituting the values of u and v in (2) , weget

$$\left.\begin{aligned}X(a,b)&=a^3+399b^3-57ab^2-63a^2b-1\\ Y(a,b)&=\frac{1}{10}[-12a^3+3952b^3+684ab^2-624a^2b-10]\end{aligned}\right\}$$ .....(12)

As our interest is on finding integer solutions, we choose a and b suitably so that the values of u and v are in integers.

Replace a by 5A and b by 5B in (4) and (12), the corresponding integral solutions of (1) are,

$X(A,B)=125A^3+49875B^3-7125AB^2-7875A^2B-1$
$Y(A,B)=-6A^3+49400B^3+8550AB^2-7800A^2B-1$
$Z(A,B)=25A^2+475B^2$

PROPERTIES

- $Y(A,1)-6cp_{5,A}-cp_{6,A}-T_{104,a}\equiv 2A(Mod\,7)$
- $Y(A,1)+9OH_A+7800T_{4,A}\equiv 6(Mod\,7)$
- $T_{52,A}-Z(A,1)\equiv -4A(Mod\,5)$
- $X(A,1)+18p_A^4+7791T_{4,a}\equiv 1(Mod\,3)$
- $6cp_{6,A}+7800pr_A+X(A,1)\equiv 4(Mod\,5)$
- $6cp_{30,A}-X(A,1)+95cp_{6,A}-T_{15752,A}\equiv 6(Mod\,7)$
- $X(1,B)-49400cp_{6,B}-1900HD_B\equiv 9(Mod\,10)$

Coclusion

To conclude, we may search for other patterns of solutions to (1) along with their properties.

Acknowledgement

The finical support from the UGC, New Delhi F.MRP-5123/14(SERO/UGC) dated march 2014) for a part of this work is gratefully acknowledged

REFERENCES

Carmichael, R.D. 1959. The Theory of Numbers and Diophantine Analysis, New York, Dover,

Dickson, L.E. 2005. History of Theory of Numbers, Vol.2, Diophantine Analysis, New York, Dover

Gopalan. M.A. *et al.* 2006 On the ternary cubic Diophantine equation $x^2+y^2=2z^3$ Advances in theoretical and applied Mathematics Vol.1, No.3, Pp.227-231

Gopalan, M.A. *et al.* 2007. On Ternary cubic Diophantine Equations $X^2-Y^2=Z^3$, Actaciencia Indica, Vol.XXXIIIM (No.3) pg.705-707.

Gopalan, M.A. *et al.* 2008. Integral solutions of Ternary cubic Diophantine Equations $X^2+Y^2+4N=ZXY$, pure and *Applied Mathematics Sciences*, Vol. LXVII (No.1-2): 107-111

Gopalan. M.A. *et al.* 2008. Integral solutions of Ternary cubic Equation $X^2-XY+Y^2=(K^2-2K+4)Z^3$, pacific *Asian journal of Mathematics*, Vol.2,(No.1-2), pg. 91-96.

Gopalan, M.A. *et al.* 2008, Integral solutions of $X^2-aY^2=(a-1)Z^3,(a>1$ and are a square free), Impact *J. Sci.Tech*., Vol.2 (4) pg.201-204.

Gopalan, M.A. *et al*. 2008. Ternary cubic Diophantine Equation $2^{2a-1}(X^2+Y^2)=Z^3$ Actaciencia Indica, Vol.XXXIVM, (No.3) pg.113-137.

Gopalan, M.A., *et al.* 2010. Integral solutions of $X^3+Y^3+8K(X+Y)=(2K+1)Z^3$ Bulletin of pure and *Applied Sciences*, Vol.29E(No.1), 95-99

Gopalan, M. A. *et al.* 2010. Integral solutions of $X^2-Y^2+XY=(m^2-5n^2)Z^3$, *Antarctica J.math.,*7(1), 63-67

Gopalan.M.A. *et al*, 2010, On the Equation $X^2-Y^2+XY=(n^2+4n-1)Z^3$, Bulletin of pure and *Applied sciences*, Vol.29E, Issue 2 pg.231-235.

Gopalan, M.A. *et al.* 2010. Integral solutions of ternary cubic Equation $X^2+Y^2-XY+2(X+Y+Z)=Z^3$, Antarctica *Journal of Mathematics*, Vol. 7,(No.4), 455-460.

Gopalan, M.A. *et al.* 2010. Integral solutions of Ternary cubic Equation $X^3+Y^3+4Z^2=3XY(X+Y)$, Antarctica *Journal of Mathematics*, 7(3), 311-315.

Gopalan, M.A. *et al.* 2010. On the ternary cubic Equation $Y^2+GZ^2=(K^2+G)X^3$, Impact *J. sci. Tec.,* Vol.4 No.4, 117-123.

Gopalan, M.A. *et al*. 2010. "On the Ternary Cubic Diophantine Equation $y^2+gz^2=(K^2+g)z^3$ *impact. J. Sci. Tech*., Vol.4, No.4, Pg117-23. 2010.

Gopalan,.M.A. *et al.* 2013. K., On the ternary cubic Diophantine equation $X^2+Y^2-XY=Z^3$, *Bessel J.Math*., 3(2), 119-123.

Modrell. L.J. 1969. Diophantine Equations, Academic press, NewYork, Dover.

******