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RESEARCH ARTICLE

FIVE DIMENSIONAL NON-VACUUM CYLINDRICALLY SYMMETRIC SOLUTIONS IN *F(R)* THEORY OF GRAVITY

[*,1]Ladke, L. S., [2]Mishra R. D. and [3]Hiwarkar R. A.

[1]Nilkanthrao Shinde Science and Arts College, Bhadrawati, India
[2]Shriram commerce and science college, Kurkheda, India
[3]Guru Nanak Institute of Engineering & Technology, Nagpur, India

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ABSTRACT

In this paper a cylindrically symmetric solutions in *f(R)* theory of gravity has been obtained by using perfect fluid. We investigate two solutions. The first solution corresponds to constant curvature and the second solution provides non-constant curvature. To find the pressure and density we used equation of state parameter.

INTRODUCTION

In recent years, researchers show their interest in modified theory of gravity. Among all modified theory, *f(R)* theory is intensively studied due to its vast application in astrophysics and space science. This theory provides a very natural gravitational alternative to problems of dark matter and dark energy (Nojiri and Odintsov, 2007). From the history, universe occupied 2\3 of dark energy, 1\3 of dark matter and other component known as baryon matter (Spergel *et al*., 2007). Due to this matter our universe is expanding with an accelerating rate (Reiss *et al*., 1998). It is interesting to note that the cosmological constant was introduced by Einstein as a alternative to dark matter and dark energy, but later he realized that it is biggest blander of his life. To resolve such issue several modification are proposed by numbers of authors (Tiberiu Harko *et al*., 2011; Ross, 1972; Brans and Dicke, 1961; Rafael Ferraro.: "f(R) and f(T) theories of modified gravity, 2012). Among all modified theory of gravity, we focus on *f(R)* theory of gravity because *f(R)* models consist of higher order curvature invariants as functions of the Ricci scalar. The unification of early time inflation and late time acceleration beprovided by using viable *f(R)* gravity models (Nojiri and Odintsov, 2008). The clarification of cosmic acceleration is obtained just by introducing the term 1/R which is necessary at small curvatures. Due to its cosmologically important *f(R)* models, this theory is considered to be most suitable. By considering higher order curvature term there does not exit any singularity in *f(R)* theory of gravity (Nojiri and Odintsov, 2008). The *f(R)* gravity can also beunderstands by using some interesting review (Shin'ichiNojiri and Sergei D. Odintsov, 2006; ChangjunGao and You-Gen Shen, 2016; Antonio De Felice and Shinji Tsujikawa, 2010). Many authors studied*f (R)* theory of gravity in different contexts. Sharif and Shamir studied studied plane symmetric solutions in metric *f(R)* gravity (Sharif and Shamir, 2010). Bianchi types I and V cosmological model for vacuum and non-vacuum cases are also solved by Sharif and Shamir (Sharif and Shamir, 2009; Sharif and Farasat Shamir, 2010). M. Farasat Shamir explored static plane symmetric vacuum solutions (Sharif and Farasat Shamir, 2009) in *f(R)* gravity with the assumption of constant scalar curvature which may be zero or non-zero. The spherical symmetric solution is the first nearby solution to the nature. Spherically symmetric vacuum solutions in *f(R)* theory are obtained by Multamaki and Vilja (Multam¨aki and Vilja, 2006). Also spherically symmetric solutions of *f(R)* theories of gravity via the Noether symmetry approach are studied by Capozziello *et*

****Corresponding author:*** **Ladke, L. S.**
Nilkanthrao Shinde Science and Arts College, Bhadrawati, India.

al. (2007). Hollenstein and Lobo (2008) explored exact solutions of static spherically symmetric space-times in *f(R)* gravity coupled to non-linear electrodynamics. Chakrabarti & Banerjee (2014) obtained a constant curvature spherically symmetric vacuum collapse in *f(R)* theory of gravity. Cylindrical symmetry is the next closest approach to universe after spherical symmetry. Cylindrically symmetric vacuum solutions in *f(R)* theory gravity explored by Azadi *et al*. (2008). Momeni and Gholizade (2009) extended cylindrically symmetric solutions in a more general way. Cylindrically symmetric vacuum and non-vacuum solutions have also been investigated in *f(R)* theory gravity (Momeni, 2010). Capozziello *et al*. (2008) have shown that dust matter and dark energy phases can be achieved by the exact solution derived from a power law *f(R)* cosmological model. Static cylindrically symmetric interior solutions in metric *f(R)* gravity have been studied by Sharif and Arif (2012). Sharif *et al*. (2012) explained the energy distribution and non-vacuum cylindrically symmetric solutions using the assumption of constant curvature. Assuming the constant and non-constant curvature solution M. Farasat Shamir (2014) studied dust static and cylindrically symmetric solutions in *f(R)* theory of gravity.

Study of higher dimensional cosmological model in *f(R)* theory of gravity is an important discussion in the field of relativity. The source of higher dimensional data comes from string theory that these theories require extra dimensions of space-time for their mathematical consistency. In case of bosonic-string theory space-time is 26-dimensional while in case of superstring theory space-time would be 10-dimensional, and in super gravity theory it is 11-dimensional. The concept of five-dimensional space-timehas been introduced by Kaluza and Kelin (Kaluza, 1921; Kelin, 1926) which unified gravitation with electromagnetic interaction. Outstanding review of higher dimensional space-time has been provided by Wesson (Wesson, 1983; Wesson, 1984) by studying several aspects in variable mass theory and biometric theory of relativity. Higher dimensional cosmological models play a vital role in many aspects of early stage of cosmological problems. The study of higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution as observed today. With this motivation, in this paper, we have studied five dimensional non-vacuumcylindrically symmetric solutions by using assumption of constant and non-constant scalar curvature in *f(R)* theory of gravity.To find pressure and energy density we have used equation of state parameter.

The action for $f(R)$ theory of gravity are given by

$$S=\int\left(\frac{1}{16\pi G}f(R)+L_m\right)\sqrt{-g}\;d^5x, \qquad (1)$$

Where $f(R)$ is general function of Ricci scalar R and L_m is the matter Lagrangian.

Now by varying the action S with respect to g_{ij}, we obtain the field equations in $f(R)$ theory of gravity as

$$F(R)R_{ij}-\frac{1}{2}f(R)g_{ij}-\nabla_i\nabla_j F(R)+g_{ij}\Box F(R)=kT_{ij},(i,j=1,2,3,4,5) \qquad (2)$$

Where $F(R)\equiv\frac{df(R)}{dR}$, $\Box\equiv\nabla^i\nabla_i$

With ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.

If we take $f(R)=R$, the field equation (2) in *f(R)* theory of gravity reduce to the field equation of general theory of relativity which is propose by Einstein.

Contracting the above field equations (2), we have

$$F(R)R-\frac{5}{2}f(R)+4\Box F(R)=kT \qquad (3)$$

For non-vacuum, we have

$$F(R)R-\frac{5}{2}f(R)+4\Box F(R)=8\pi T \qquad (4)$$

From (4), we get

$$f(R)=\frac{2}{5}\left[-8\pi T+4\Box F(R)+F(R)R\right] \qquad (5)$$

Using equations (2) and (5), the field equations take the form

$$\frac{F(R)R_{ij}-\nabla_i\nabla_j F(R)-8\pi T_{ii}}{g_{ij}}=\frac{1}{5}[F(R)R-\Box F(R)-8\pi T] \qquad (6)$$

It follows that the equation (6) is not depend on the index i

Metric & the Field equations

Theline element of cylindrically symmetric space-time

$$ds^2=A^2dt^2-dr^2-B^2\left(d\phi^2+\sigma^2dz^2\right)-c^2d\psi^2 \qquad (7)$$

Where A, B and C are functions of radial co-ordinate r and σ is an arbitrary constant.

The Ricci scalar is

$$R=4\left[\frac{\ddot{B}}{\dot{B}}+\frac{\ddot{C}}{2C}+\frac{\ddot{A}}{2A}+\frac{\dot{B}^2}{2B^2}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{2AC}\right] \qquad (8)$$

Where dot denotes derivative with respect to r

The stress energy tensor for perfect fluid is given by

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij} \qquad (9)$$

The matter density is given by the scalar function ρ and Pressure p

Where $u_i=\delta^5{}_i$ and u_i is four velocity

Equation (6) can be express as

$$K_i = \frac{F(R)R_{ij} - \nabla_i \nabla_j F(R) - 8\pi T_{ij}}{g_{ij}}$$

(10)

This equation is independent of i and hence $K_i - K_j = 0$ for all i and j.

From above equation, we have

$K_5 - K_1 = 0$, gives

$$\frac{2\dot{A}\dot{B}F}{AB} + \frac{\dot{A}\dot{C}F}{AC} - \frac{2\ddot{B}F}{B} - \frac{\ddot{C}F}{C} + \frac{\dot{A}\dot{F}}{A} - \ddot{F} - \frac{K}{A^2}(\rho + p) = 0 \quad (11)$$

$K_5 - K_2 = K_5 - K_3 = 0$, gives

$$\frac{\ddot{A}F}{A} + \frac{\dot{A}\dot{B}F}{AB} + \frac{\dot{A}\dot{C}F}{AC} + \frac{\dot{A}\dot{F}}{A} - \frac{\ddot{B}F}{B} - \frac{\dot{B}^2 F}{B^2} - \frac{\dot{B}\dot{C}F}{BC} - \frac{\dot{B}\dot{F}}{B} - \frac{K}{A^2}(\rho + p) = 0 \quad (12)$$

Similarly, $K_5 - K_4 = 0$ yield

$$\frac{\ddot{A}F}{A} + \frac{2\dot{A}\dot{B}F}{AB} + \frac{\dot{A}\dot{F}}{A} - \frac{\ddot{C}F}{C} - \frac{2\dot{C}\dot{B}F}{CB} - \frac{\dot{C}\dot{F}}{C} - \frac{K}{A^2}(\rho + p) = 0 \quad (13)$$

For the simplicity we take $A = 1$ in above equation we get

$$\frac{2\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{F}}{F} + \frac{K}{F}(\rho + p) = 0 \quad (14)$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} + \frac{K}{F}(\rho + p) = 0 \quad (15)$$

$$\frac{\ddot{C}}{C} + \frac{2\dot{C}\dot{B}}{CB} + \frac{\dot{C}\dot{F}}{CF} + \frac{K}{F}(\rho + p) = 0$$

(16)

Constant curvature solutions

Subtracting equation (15) from equation (14), we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{F}}{F} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{F}}{BF} = 0$$

(17)

Subtracting equation (16) from equation (15), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{C}}{C} - \frac{\dot{C}\dot{B}}{CB} - \frac{\dot{C}\dot{F}}{CF} = 0 \quad (18)$$

To solve the field equations, we assume

$$F(R) \propto f_0 R^m \quad (19)$$

f_0 and m are arbitrary constant

By using equation (19), Equation (17) becomes

$$m(m-1)\frac{\dot{R}^2}{R^2} + m\left(\frac{\ddot{R}}{R} - \frac{\dot{B}\dot{R}}{BR}\right) + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{B}^2}{B^2} = 0 \quad (20)$$

By taking the constraint $m(m-1) = 0$ and using equation (8) in equation (20)

We get the solution

$$B(r) = e^{k_1 r} \text{ and } C(r) = e^{k_2 r} \quad (21)$$

Where k_1 and k_2 are arbitrary constants

The metric takes the form

$$ds^2 = dt^2 - dr^2 - e^{2k_1 r}\left(d\phi^2 + \sigma^2 dz^2\right) - e^{2k_2 r} d\psi^2 \quad (22)$$

and due to constraint there arises two cases

Case I

When $m = 0$ in $F(R) = f_0 R^m$

$F(R) = f_0$ This correspond to $f(R) = f_0 R + k_3$

Where k_3 is integration constant

For $k_3 = 0$ and $f_0 = 1$ the solution correspond to GR.
It is observed that the constant curvature solution in *f(R)* theory of gravity corresponds to already available solutions in general relativity.

The energy density and pressure calculated by using equation of state parameter $\omega = \frac{p}{\rho}$

From equation (15), we get the constant values of function of Ricci scalar *f(R)* i.e.

$$f(R) = f_0\left(6k_1^2 + 2k_2^2 + 4k_1k_2\right) + k_3$$
$$R = \left(6k_1^2 + 2k_2^2 + 4k_1k_2\right)$$
$$f(R) = 2f_0c_1 + k_3 \ (f_0 < 0) \quad (23)$$

$$R = 2c_1 \quad (24)$$

$$\rho = \frac{-k_1}{K(1+\omega)}(k_2 + 2k_1)f_0 \ (f_0 < 0) \quad (25)$$

$$p = \frac{-k_1\omega}{K(1+\omega)}(k_2 + 2k_1)f_0 \ (f_0 < 0) \quad (26)$$

Where $c_1 = \left(3k_1^2 + k_2^2 + 2k_1k_2\right)$

For physical solution, we have taken f_0 as negative.

Case II

When $m = 1$, the function of Ricci scalar calculated as

$$F(R) = f_0 R$$

$$f(R) = 2 f_0 c_1^2 + k_4 \tag{27}$$

Where $c_1 = \left(3k_1^2 + k_2^2 + 2k_1 k_2\right)$ and k_4 is an integration constant

From (15), the energy density and pressure turned out to be

$$\rho = \frac{-k_1}{K(1+\omega)} f_0 \left(k_2 + 2k_1\right)\left(6k_1^2 + 2k_2^2 + 4k_1 k_2\right)$$

$$\rho = \frac{-2k_1}{K(1+\omega)} f_0 c_1 \left(k_2 + 2k_1\right) \left(f_0 < 0\right) \tag{28}$$

$$p = \frac{-2k_1 \omega}{K(1+\omega)} f_0 c_1 \left(k_2 + 2k_1\right) \left(f_0 < 0\right) \tag{29}$$

Where $c_1 = \left(3k_1^2 + k_2^2 + 2k_1 k_2\right)$

In this case also we have taken f_0 as negative, to get the physical solution.

Non-constant curvature Solution

Here considering the solution of equation (20) in power law form

$$B(r) = \left(k_3 r + k_4\right)^n \text{ and } C(r) = \left(k_5 r + k_6\right)^l \tag{30}$$

Where k_3, k_4, k_5, k_6 are constants and n, l are integers

Here $k_4 = 0$ and $k_6 = 0$ we get

$$B(r) = \left(k_3 r\right)^n \text{ and } C(r) = \left(k_5 r\right)^l \tag{31}$$

Using equation (31) in equation (14),(15),(16), we get

$n = 1$ and $l = -1$

Using above condition in equation (20) we get

$$2m^2 + 2m - 1 = 0 \tag{32}$$

The metric takes the form

$$ds^2 = dt^2 - dr^2 - k_3^2 r^2\left(d\phi^2 + \sigma^2 dz^2\right) - k_5^{-2} r^{-2} d\psi^2 \tag{33}$$

From equation (9) we get the Ricci scalar without constant

$$R = \frac{2}{r^2} \tag{34}$$

Equation (32) has two roots and for two different roots two cases are arises

Case III

From equation (32), we obtained the first root which is in following form

$$m = \frac{-1+\sqrt{3}}{2}$$

Corresponding to this root, the value of *f(R)* is

$$F(R) = f_0 R^{\frac{-1+\sqrt{3}}{2}} \tag{35}$$

Integrating equation (35) and using the value of equation (34), we get

$$f(R) = \hat{f}\left(\frac{2}{r^2}\right)^{\frac{-1+\sqrt{3}}{2}} + k_7 \tag{36}$$

Where $\hat{f} = \dfrac{2f_0}{-1+\sqrt{3}}$ and k_7 is integration constant

By using equation (16), the energy density and pressure of the universe becomes

$$\rho = \frac{-f_0}{K(1+\omega)}\left(-1+\sqrt{3}\right)\frac{2^{\frac{-1+\sqrt{3}}{2}}}{r^{1+\sqrt{3}}} \left(f_0 < 0\right) \tag{37}$$

$$p = \frac{-f_0 \omega}{K(1+\omega)}\left(-1+\sqrt{3}\right)\frac{2^{\frac{-1+\sqrt{3}}{2}}}{r^{1+\sqrt{3}}} \left(f_0 < 0\right) \tag{38}$$

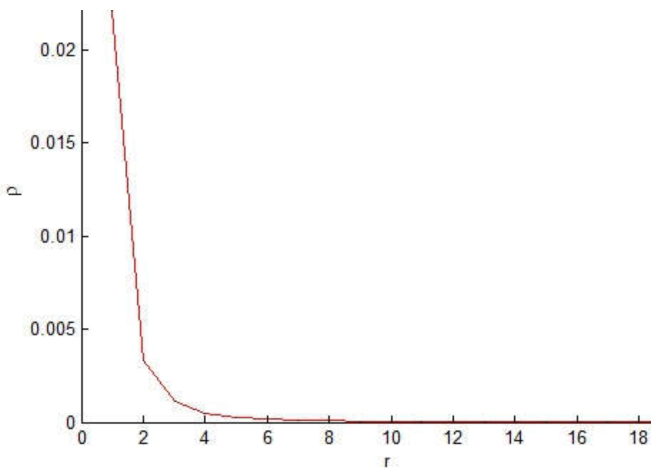

Figure 1. Graph of energy density versus radial coordinates

Case IV

From equation (32), we obtained the second root which is in following form

$$m = \frac{-1-\sqrt{3}}{2}$$

The corresponding value of F(R) becomes

$$F(R) = f_0 R^{\frac{-1-\sqrt{3}}{2}} \tag{39}$$

Integrating equation (39) andusing the values of equation (34), we get

$$f(R) = \check{f}\left(\frac{2}{r^2}\right)^{\frac{-1-\sqrt{3}}{2}} + k_8 \tag{40}$$

Where $\check{f} = \frac{2f_0}{-1-\sqrt{3}}$ and k_8 is integration constant.

$$\rho = \frac{-f_0}{K(1+w)} 2^{\frac{-1-\sqrt{3}}{2}} \left(\frac{-1-\sqrt{3}}{r^{1-\sqrt{3}}}\right), \quad (f_0 < 0)$$

$$p = \frac{-f_0 \omega}{K(1+w)} 2^{\frac{-1-\sqrt{3}}{2}} \left(\frac{-1-\sqrt{3}}{r^{1-\sqrt{3}}}\right), \quad (f_0 < 0)$$

Where ρ and p are the density and pressure of the universe.

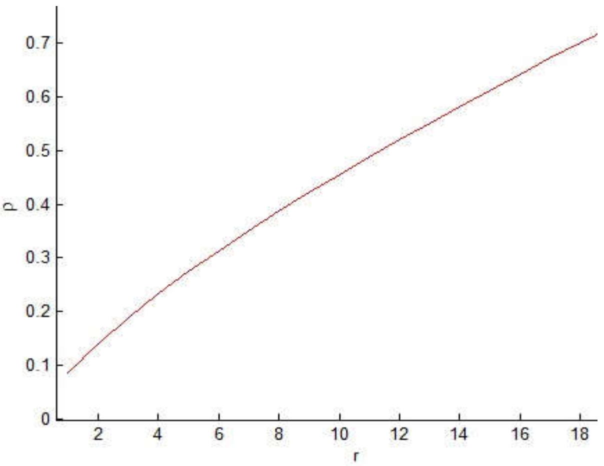

Figure 2. Graph of energy density versus radial coordinates

DISCUSSION AND CONCLUSION

In this paper we investigate five dimensional non-vacuum cylindrical symmetric solutions in *f(R)* theory of gravity. We find both constant and non-constant curvature solutions. Also we have used perfect fluid case to explore the solutions and equation of state parameter to find the pressure and energy density. We have found the constant curvature solutions in two cases and in both cases it is observed that the Ricci scalar R, function of Ricci scalar *f(R),* matter density ρ and pressure p, all are constant. We have also obtained non-constant curvature solution in two cases. In first case, as r approaches to infinity, matter density ρ goes to zero. In second case density increases with increase in r. In both the cases *f(R)* is having negative power of curvature which supports the current cosmic acceleration. For both the solutions we have calculated pressure p which is a function radial co-ordinate r. The results obtained here are similar to results obtained in (27).

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